Enhancements of the bisimulation proof method:
some results, some open problems

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This talk

Enhancements of the bisimulation proof method

- Motivations
- Results and Examples
- Open problems

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Equality on processes, coinductively

Bisimulation:

A relation $\mathcal{R}$ s.t.

$$\begin{array}{ccc} P & \mathcal{R} & Q \\ \alpha\downarrow & & \downarrow\alpha \\ P' & \mathcal{R} & Q' \end{array}$$

Bisimilarity ($\sim$) :

$$\bigcup \{\mathcal{R} : \mathcal{R} \text{ is a bisimulation } \}$$

Hence:

$$\frac{x\ \mathcal{R}\ y \qquad \mathcal{R} \text{ is a bisimulation}}{x \sim y}$$

(bisimulation proof method)

Enhancements of the method: an example

The perfect-firewall equation in Ambients

P: a process with n not free in it

$$\nu n \;\; n\langle P\rangle \sim 0$$

Proof: **Let's find a bisimulation...**

Is this a bisimulation?

$$\mathcal{R} \triangleq \{ (\nu n\ n\langle P \rangle , 0) \}$$

Is this a bisimulation?

$$\mathcal{R} \triangleq \{ (\nu n\ n\langle P \rangle\ ,\ 0) \}$$

No!

Suppose $n\langle P \rangle \xrightarrow{\texttt{enter\_}k\langle Q \rangle} n\langle P \rangle$

(the loop: simplifies the example, not necessary)

$$\begin{array}{ccc} \nu n\ n\langle P \rangle & \mathcal{R} & 0 \\ \texttt{enter\_}k\langle Q \rangle \downarrow & & \downarrow \texttt{enter\_}k\langle Q \rangle \\ k\langle Q \mid \nu n\ n\langle P \rangle \rangle & \not\mathcal{R} & k\langle Q \rangle \mid 0 \end{array}$$

Try again...

Is this a bisimulation?

$$\mathcal{R} \triangleq \{ (\nu n\ n\langle P\rangle\ ,\ 0) \} \cup_{k,Q} \{ (k\langle Q \mid \nu n\ n\langle P\rangle \rangle\ ,\ k\langle Q\rangle \mid 0) \}$$

Is this a bisimulation?

$$\mathcal{R} \triangleq \{ (\nu n\ n\langle P\rangle\ ,\ 0) \} \cup_{k,Q} \{ (k\langle Q \mid \nu n\ n\langle P\rangle\ \rangle\ ,\ k\langle Q\rangle \mid 0) \}$$

No!

Suppose $Q = h\langle \text{out}\ k.\ R\rangle \mid Q'$

$$k\langle Q \mid \nu n\ n\langle P\rangle\ \rangle \quad \mathcal{R} \quad k\langle Q\rangle \mid 0$$

$$\downarrow \qquad\qquad \downarrow$$

$$k\langle Q' \mid \nu n\ n\langle P\rangle\ \rangle \mid h\langle R\rangle \quad \not\mathcal{R} \quad k\langle Q'\rangle \mid h\langle R\rangle \mid 0$$

Try again...

Is this a bisimulation?

$$\mathcal{R} \triangleq \{\, (\nu n\ n\langle P\rangle\ ,\ 0)\ \} \cup_{k,Q} \{\, (k\langle Q \mid \nu n\ n\langle P\rangle\ \rangle\ ,\ k\langle Q\rangle \mid 0)\ \}$$

Also:

Suppose $Q = \mathtt{in}\, h.\, Q'$

$$\begin{array}{ccc} k\langle Q \mid \nu n\ n\langle P\rangle\ \rangle & \mathcal{R} & k\langle Q\rangle \mid 0 \\ \mathtt{enter}\_h\langle R\rangle \downarrow & & \downarrow \mathtt{enter}\_h\langle R\rangle \\ h\langle R \mid k\langle Q' \mid \nu n\ n\langle P\rangle\ \rangle\ \rangle & \not\mathcal{R} & h\langle R \mid k\langle Q'\ \rangle\ \rangle \mid 0 \end{array}$$

Try again...

The bisimulation:

$$\mathcal{R} \triangleq \bigcup C \text{ is a static contexts}$$

$$\{(S,T) \;:\; S \sim C[\nu n\ n\langle P\rangle\ ] \quad T \sim C[0]\ \}$$

$$C ::= k\langle C\rangle \mid P \mid C \mid \nu a\, C \mid [\,]$$

We started with the **singleton** relation

$$\{(\nu n\ n\langle P\rangle\ ,\ 0)\}$$

The added pairs: **redundant**? (derivable, laws of $\sim$)

Can we work with relations smaller than bisimulations?

Advantage: fewer and simpler bisimulation diagrams

Redundant pairs

What we would like to have:

$$\begin{array}{ccc} P & \mathcal{R} & Q \\ \alpha\downarrow & & \downarrow\alpha \\ P' & \mathcal{R} \cup \{\textbf{redundant pairs}\} & Q' \end{array} \quad \text{implies } \mathcal{R} \subseteq \sim$$

$\mathcal{R}$: less work, simpler to find

Notation

$$\mathcal{R} \rightarrowtail \mathcal{S} \triangleq \begin{array}{ccc} P & \mathcal{R} & Q \\ \alpha\downarrow & & \downarrow\alpha \\ P' & \mathcal{S} & Q' \end{array}$$

Up-to techniques: examples

– **Rules for transitivity of $\sim$ (up-to $\sim$)** [Milner]

$\mathcal{R} \rightarrowtail \sim \mathcal{R} \sim$ implies $\mathcal{R} \subseteq \sim$

diagram :

$$\begin{array}{ccccccc} P & & & & \mathcal{R} & & Q \\ \alpha\downarrow & & & & & & \downarrow\alpha \\ P' & \sim & P'' & \mathcal{R} & Q'' & \sim & Q' \end{array}$$

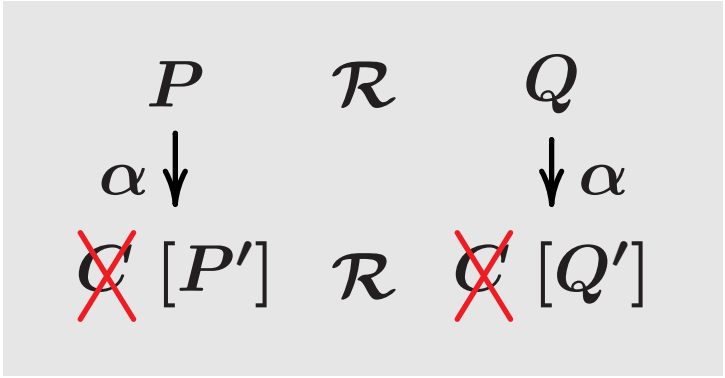
Up-to techniques: examples

- Rules for transitivity of $\sim$ (up-to $\sim$)
- **rules for substitutivity of $\sim$ (up-to context)** [Sangiorgi]

$$\mathcal{C}(\mathcal{R}) \triangleq \{(C[P], C[Q]) \ : \ P \ \mathcal{R} \ Q\}$$

$\mathcal{R} \rightarrowtail \mathcal{C}(\mathcal{R})$ implies $\mathcal{R} \subseteq \sim$

diagram :



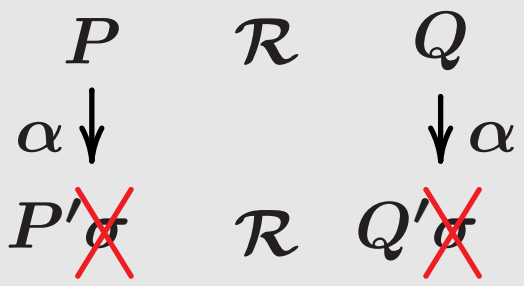
Up-to techniques: examples

- Rules for transitivity of $\sim$ (up-to $\sim$)
- rules for substitutivity of $\sim$ (up-to context)
- **rules for invariance of $\sim$ under injective substitutions (up-to injective substitutions)**

$\mathbf{Inj}(\mathcal{R}) \triangleq \{(P\sigma, Q\sigma) \ : \ P \ \mathcal{R} \ Q \ , \ \sigma \text{ injective on names}\}$

$\mathcal{R} \rightarrowtail \mathbf{Inj}(\mathcal{R})$ implies $\mathcal{R} \subseteq \sim$

diagram :



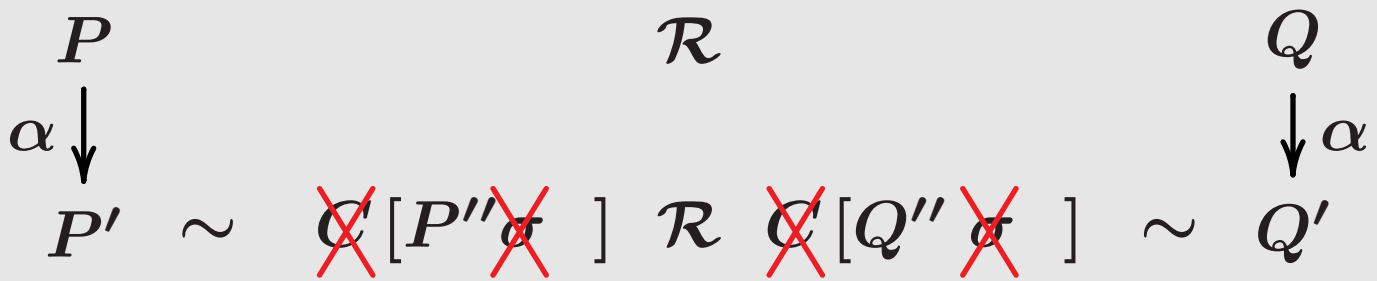
σ: an injective function

implies $\mathcal{R} \subseteq \sim$

Composition of techniques

$\mathcal{R} \rightarrowtail \sim C[\mathbf{Inj}(\mathcal{R})] \sim$ implies $\mathcal{R} \subseteq \sim$

diagram :

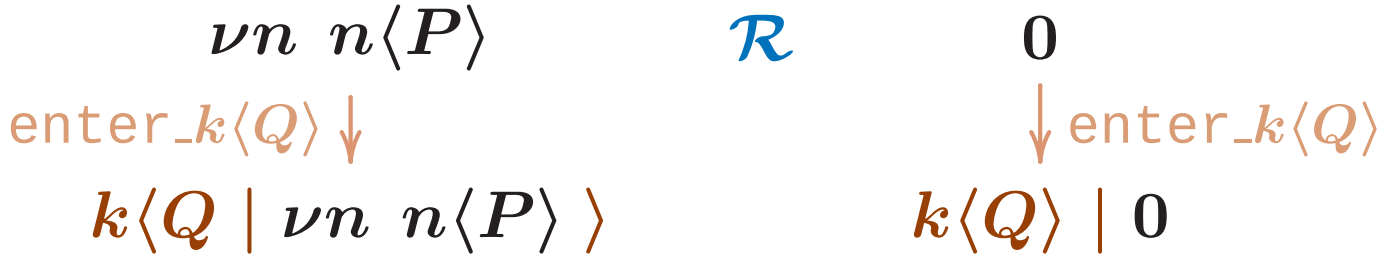


More sophistication $\Rightarrow$

- **more powerful technique**
- **harder soundness proof for the technique**

Proof of the firewall, composition of up-to techniques

We can prove $\nu n\ n\langle P\rangle \sim 0$ using the singleton relation



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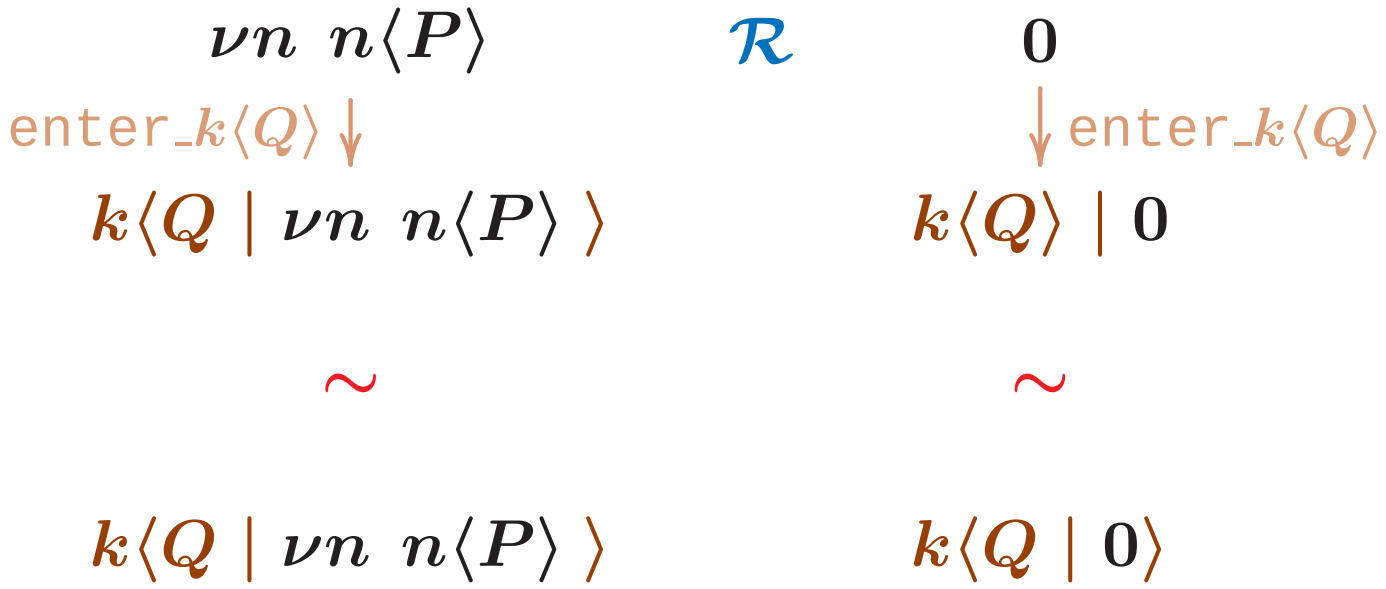
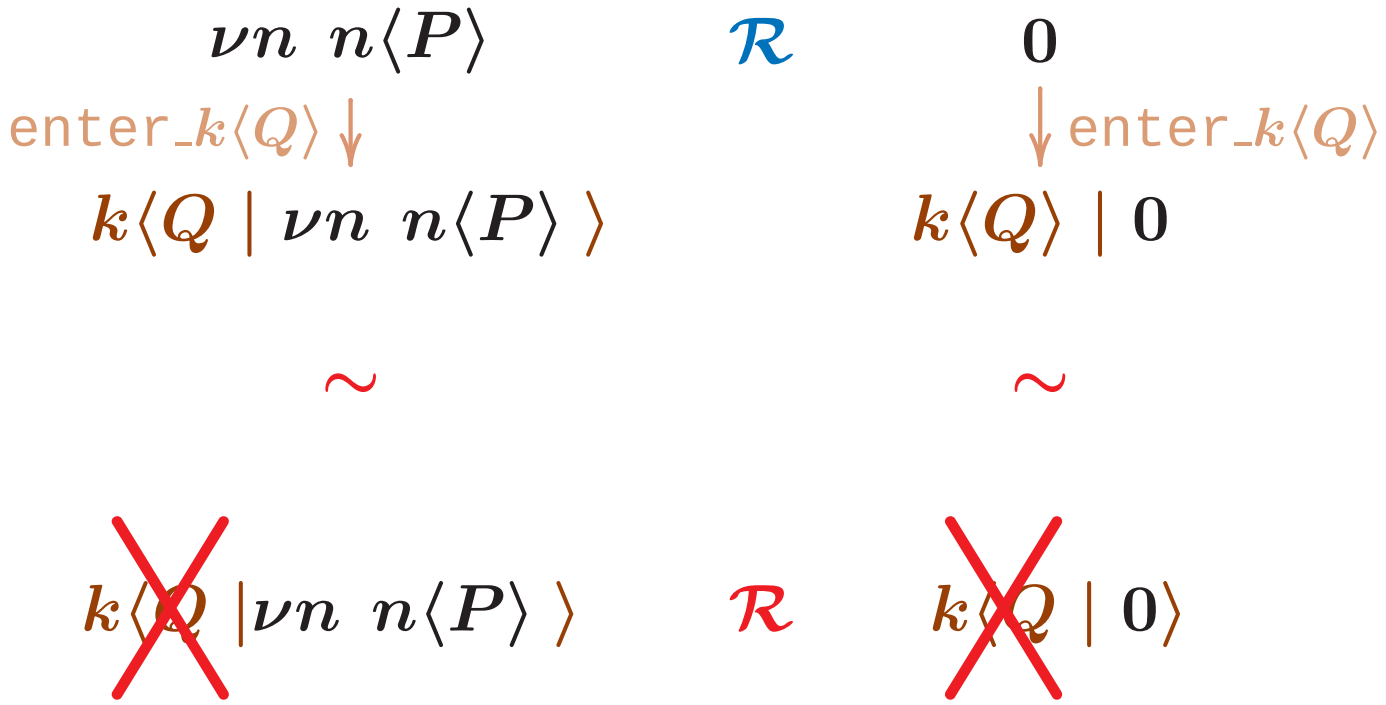


Proof of the firewall, composition of up-to techniques

We can prove $\nu n\ n\langle P\rangle \sim 0$ using the singleton relation



[Zappa-Nardelli, Merro, JACM, to appear]

"up-to $\sim$" **and** "up-to context"

(full proof also needs up-to injective substitutions)

Conclusions, part I

- **Enhancements of the bisimulation proof methods: extremely useful**
  - **essential** in π-calculus-like languages, higher-order languages
- **Various forms of enhancement ("up-to techniques") exist**
  - composition of techniques
- **Proofs of soundness of these techniques may be non-trivial**
  - separate ad hoc proofs for each technique

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Redundant pairs: first attempt

$\mathcal{S} \triangleq$ a set of inference rules valid for $\sim$

(P, Q) **redundant** in $\{(P, Q)\} \cup \mathcal{R}$ if

$$\mathcal{S} \quad \frac{\mathcal{R} \subseteq \sim}{P \sim Q}$$

Sound ? i.e.:

$$\begin{array}{ccc} P & \mathcal{R} & Q \\ \alpha\downarrow & & \downarrow\alpha \\ P' & \mathcal{R} \cup \{\textbf{redundant pairs}\} & Q' \end{array} \quad \text{implies } \mathcal{R} \subseteq \sim \quad ?$$

False!

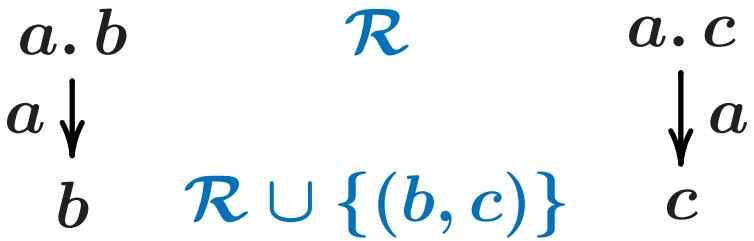
Counterexample (in CCS)

$$\mathcal{S} \triangleq \frac{a.P \sim a.Q}{P \sim Q}$$

$$\mathcal{R} \triangleq \{(a.b, a.c)\}$$

(b, c) redundant in $\mathcal{R} \cup \{(b, c)\}$

$$\mathcal{S}\ \frac{\mathcal{R} \subseteq \sim}{b \sim c}$$



but $a.b \not\sim a.c$

Example: up-to context that fails

$$P \;:=\; f(P) \;\Big|\; a.\,P \;\Big|\; 0$$

$$\frac{P \xrightarrow{a} P' \qquad P' \xrightarrow{a} P''}{f(P) \xrightarrow{a} P''}$$

Bisimulation is a congruence, yet:

$$\begin{array}{ccc} a.0 & \mathcal{R} & a.a.0 \\ a\downarrow & & \downarrow a \\ 0 & \sim f(a.\,0) & f(a.\,a.\,0) \sim \; a.0 \end{array}$$

Example: up-to context that fails

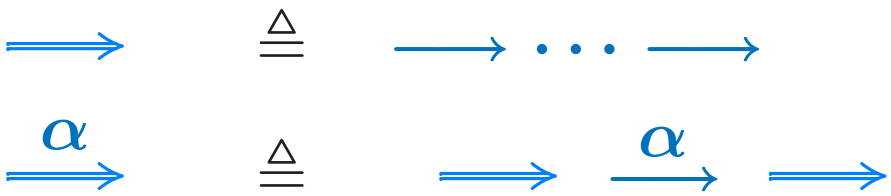
$$P \;:=\; f(P) \;\Big|\; a.\, P \;\Big|\; 0$$

$$\frac{P \xrightarrow{a} P' \qquad P' \xrightarrow{a} P''}{f(P) \xrightarrow{a} P''}$$

Bisimulation is a congruence, yet:

$$\begin{array}{ccccccc} a.0 & & \mathcal{R} & & a.a.0 \\ a\downarrow & & & & \downarrow a \\ 0 & \sim & \not{f}(a.\,0) & \mathcal{R} & \not{f}(a.\,a.\,0) & \sim & a.0 \end{array}$$

Weak bisimilarity ($\approx$)



Weak bisimulation

Too heavy:

$$\begin{array}{ccc} P & \mathcal{R} & Q \\ \alpha\Downarrow & & \Downarrow\alpha \\ P' & \mathcal{R} & Q' \end{array}$$

Better:

(read: $\Rightarrow$)

$$\begin{array}{ccc} P & \mathcal{R} & Q \\ \alpha\downarrow & & \Downarrow\alpha \\ P' & \mathcal{R} & Q' \end{array}$$

Example: up-to bisimilarity that fails

$\approx$ is transitive, yet:

$$\begin{array}{ccc} \tau.a.0 & \mathcal{R} & 0 \\ \downarrow & & \Downarrow \\ a.0 & & 0 \\ \approx & & \approx \\ \tau.a.0 & \mathcal{R} & 0 \end{array}$$

Conclusions, part II

- **When is a pair redundant?**
- **Needed: a general theory of enhancements of the bisimulation proof method**
  - powerful techniques
  - combination of techniques
  - easy to derive their soundness
- **A proposal: sound functions, respectful functions**

  [Sangiorgi]
- NB: all results that follow proved in Coq

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Sound functions

$\mathcal{F} : \wp(\mathcal{P} \times \mathcal{P}) \mapsto \wp(\mathcal{P} \times \mathcal{P}) :$

sound if $\mathcal{R} \rightarrowtail \mathcal{F}(\mathcal{R})$ implies $\mathcal{R} \subseteq \sim$

Each sound function: a valid enhancement

- **Are there interesting sound functions?**
- **Properties:**
  - membership easy to check?
  - nice compositionality properties?

Sound functions

$\mathcal{F} : \wp(\mathcal{P} \times \mathcal{P}) \mapsto \wp(\mathcal{P} \times \mathcal{P}) :$

sound if $\mathcal{R} \rightarrowtail \mathcal{F}(\mathcal{R})$ implies $\mathcal{R} \subseteq \sim$

Each sound function: a valid enhancement

- **Are there interesting sound functions? YES**
- **Properties:**
  - membership easy to check? **NO**
  - nice compositionality properties? **NO**

Towards an algebra of up-to techniques

$\mathcal{F}$: Relations $\mapsto \wp(\mathcal{P} \times \mathcal{P})$

respectful if $\dfrac{\mathcal{R} \subseteq \mathcal{S} \qquad \mathcal{R} \rightarrowtail \mathcal{S}}{\mathcal{F}(\mathcal{R}) \subseteq \mathcal{F}(\mathcal{S}) \qquad \mathcal{F}(\mathcal{R}) \rightarrowtail \mathcal{F}(\mathcal{S})}$

Examples:

identity	$\mathcal{I}$	$[\mathcal{I}(\mathcal{R}) = \mathcal{R}]$
constant-to $\sim$	$\mathcal{U}$	$[\mathcal{U}(\mathcal{R}) = \sim]$
closure under monadic contexts	$\mathcal{C}$	
closure under inj. substitutions	$\mathbf{Inj}$	

Proofs of respectfulness: easy

Non-example: **constant-to** $\mathcal{P} \times \mathcal{P}$

Compositionality properties

A respectful second-order function:
preserves the respectfulness of its arguments

Examples:

composition $\circ$ $\quad [\ (\mathcal{G} \circ \mathcal{F})\langle \mathcal{R} \rangle = \mathcal{G}\langle \mathcal{F}\langle \mathcal{R} \rangle \rangle\ ]$

union $\bigcup_{i \in I}$ $\quad [\ (\bigcup_{i \in I} \mathcal{F}_i)\langle \mathcal{R} \rangle \triangleq \bigcup_{i \in I} (\mathcal{F}_i \langle \mathcal{R} \rangle)\ ]$

chaining $\frown$ $\quad [\ (\mathcal{G} \frown \mathcal{F})\langle \mathcal{R} \rangle = \mathcal{G}(\mathcal{R})\ \mathcal{F}(\mathcal{R})\ ]$

Proofs of respectfulness: easy

The previous up-to techniques can be derived:

$\mathcal{U}\,\widehat{}\,\mathcal{I}\,\widehat{}\,\mathcal{U}$ = up-to $\sim$

$$\begin{array}{ccc} P & \mathcal{R} & Q \\ \alpha\downarrow & & \downarrow\alpha \\ P' & \sim\mathcal{R}\sim & Q' \end{array}$$

$\bigcup_{n>0} \underbrace{\mathcal{I}\,\widehat{}\,\cdots\,\widehat{}\,\mathcal{I}}_{n}$ = up-to transitive closure

$$\begin{array}{ccc} P & \mathcal{R} & Q \\ \alpha\downarrow & & \downarrow \\ P' & \mathcal{R}^{+} & Q \end{array}$$

Similarly we derive: $\mathcal{C}^*$ (up-to **polyadic** contexts)

$$\sim \mathcal{C}^*(\mathbf{Inj}(\mathcal{R})) \sim$$

Conclusions, part III

- **An attempt of an algebra of enhancements**
  - Minimal basic ingredients

    (identity function, constant functions, ...)
  - 2nd order functions to derive more powerful techniques
- **Sufficient to derive many techniques of practical interest**

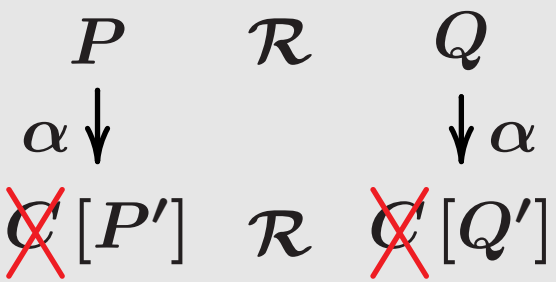
  (for strong bisimulation)
- **However**, in this theory:
  - ad hoc definitions?
  - all proofs very easy

Problem 1: Robust definition of enhancement

- **Better definition of respectfulness ?**
- **Abstract formulations of a more powerful bisimulation principle ?**
- **Generalisation to coinduction ?**
  - Partial results on coalgebras **[Lenisa, Honsell]**

Problem 2: soundness of up-to context

- **What conditions on contexts for the up-to context to be sound?**
  - Bisimulation as a congruence? i.e.:



sound iff C preserves $\sim$ **?**

- **And for respectfulness?**
  - Bisimulation as a congruence? **No!**
  - Partial answer: some behavioural conditions on contexts

    [Sangiorgi]

Problem 3: up-to context in higher-order languages

Example: λ-calculus (call-by-value/name,typed/untyped,...)

$$M \overset{\lambda R}{\Longrightarrow} N \quad \triangleq \quad M \Longrightarrow \lambda x.\, M' \quad \& \quad N = M'\{{}^{R}/_{x}\}$$

Applicative bisimulation ($\simeq$) :

$$\begin{array}{ccc} M & \mathcal{R} & N \\ {\scriptstyle \lambda R}\Downarrow & & \Downarrow{\scriptstyle \lambda R} \\ M' & \mathcal{R} & N' \end{array}$$

Theorem: $\simeq$ is a congruence

Applicative bisimulation up-to polyadic contexts

$$\begin{array}{ccc} M & \mathcal{R} & N \\ {\scriptstyle \lambda R}\Downarrow & & \Downarrow{\scriptstyle \lambda R} \\ \not{\mathcal{C}}[M_1, \ldots, M_n] & \mathcal{R} & \not{\mathcal{C}}[N_1, \ldots, N_n] \end{array}$$

implies $\mathcal{R} \subseteq \simeq$?

- Related to the problem of **compositionality** of bisimulation?
- Soundness of **limited forms of up-to context** :
  - **[Pitts 96, Lassen 98]**: typed and untyped λ-calculus, for various reduction strategies
  - **[Koutavas, Wand 06]**: a λ-calculus with references

Example of use: **[Lassen]**

Park-induction property for various fixed-point combinators (Curry, Turing, call-by-value, rec)

$$\frac{\lambda x.\, e\{\, v/f \,\} \lesssim v}{\mathtt{rec} f = \lambda x.\, e \lesssim v}$$

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Example: up-to bisimilarity that fails

$$
\begin{array}{ccc}
\tau.a.0 & \mathcal{R} & 0 \\
\downarrow & & \Downarrow \\
a.0 & & 0 \\
\approx & & \approx \\
\tau.a.0 & \mathcal{R} & 0
\end{array}
$$

Example: up-to bisimilarity that fails

$$
\begin{array}{ccc}
\tau.a.0 & \mathcal{R} & 0 \\
\downarrow & & \Downarrow \\
a.0 & & 0 \\
\\
\approx & & \approx \\
\\
\tau.a.0 & \mathcal{R} & 0
\end{array}
$$

- **Chaining** (ie: relational composition) is not respectful
- **What in place of $\approx$?**
  - **Expansion ($\lesssim$)**

    [Arun-Kumarm, Hennessy 91; Milner, Sangiorgi 92]
  - **Controlled relations** **[Pous 05]**

Example: up-to bisimilarity that fails

$$\begin{array}{ccc} \tau.a.0 & \mathcal{R} & 0 \\ \downarrow & & \Downarrow \\ a.0 & & 0 \\ \\ \approx & & \approx \\ \\ \tau.a.0 & \mathcal{R} & 0 \end{array}$$

- **Chaining** (ie: relational composition) is not respectful
- **What in place of $\approx$?**
  - **Expansion ($\lesssim$)**

    [Arun-Kum
  - **Controlled relations** [

candidate relations contain only "normal forms"

Expansion ($\lesssim$)

$P \lesssim Q$ if: $\begin{cases} \mathbf{P} \approx \mathbf{Q} \\ \mathbf{P} \text{ is more efficient than } \mathbf{Q} \end{cases}$

Definition:

1. $$\begin{array}{ccc} P & \lesssim & Q \\ {\scriptstyle\alpha}\downarrow & & \Downarrow{\scriptstyle\alpha} \\ P' & \lesssim & Q' \end{array}$$ read: ($\Rightarrow$)

2. $$\begin{array}{ccc} Q & \gtrsim & P \\ {\scriptstyle\alpha}\downarrow & & \downarrow{\scriptstyle\alpha} \\ Q' & \gtrsim & P' \end{array} \qquad \begin{array}{ccc} Q & \gtrsim & P \\ \downarrow & & \| \\ Q' & \gtrsim & P \end{array}$$ read: ($\Rightarrow$)

Examples: $P \lesssim \tau.P$

$P \not\gtrsim \tau.P$

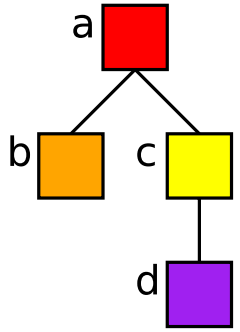
Example: correctness of an abstract machine for (Safe) Ambients

[Giannini, Sangiorgi, Valente, 04]

Nesting of ambients yields a tree

Example:

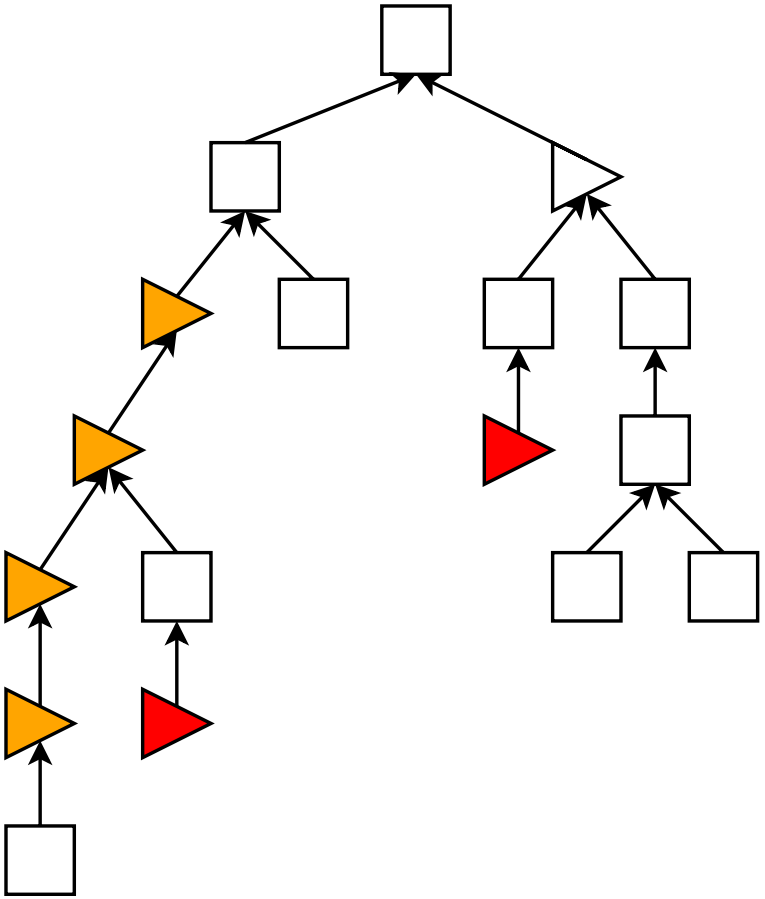
$a\langle b\langle\rangle \mid c\langle d\langle\rangle\rangle\rangle$ becomes



Movements of ambients:

- modify the tree structures
- can produce **forwarders**

The abstract machine – graphical representation



- Forwarders: common in distributed systems
- Forwarder chains
- Possible useless forwarders

Correctness proof (sketch)

– Ideally, using $\mathcal{R} \triangleq \{([\![P]\!], P)\}$

P : an Ambient term
$[\![P]\!]$: representation of P in the AM

– However:

$$[\![P]\!] \qquad \mathcal{R} \qquad P$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\nu h\,(h \rhd k \mid [\![Q(h)]\!]) \qquad \not{\mathcal{R}} \qquad Q(k)$$

– Further: in the AM there may be messages floating around (cf: non-atomicity of the implementation of Ambient basic operations)

– Indeed: the bisimulation relation needed is **very** complex.

$\mathcal{R}$ works if we use expansion:

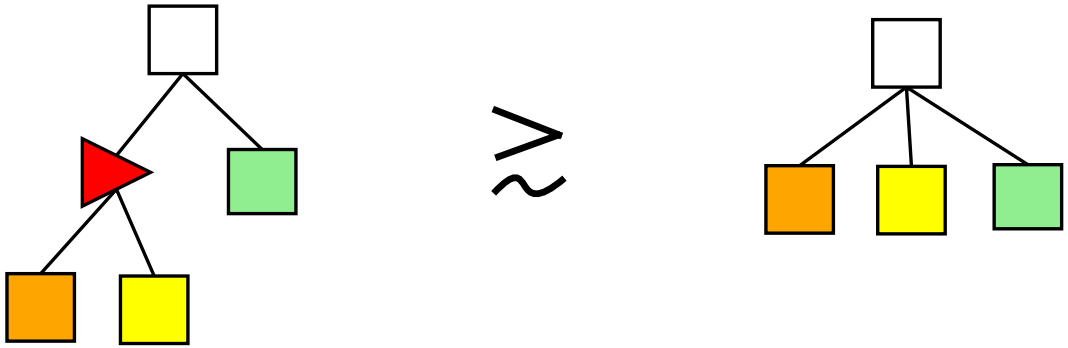
$$\begin{array}{ccc} [\![P]\!] & \mathcal{R} & P \\ \downarrow & & \downarrow \\ \nu h\,(h \triangleright k \mid [\![Q(h)]\!]) & & Q(k) \\ \gtrsim & & \approx \\ [\![Q(k)]\!] & \mathcal{R} & Q(k) \end{array}$$

Lemma: If h used only for messages in A

$$\nu h\,(h \triangleright k \mid A) \gtrsim A\{\,^{k}/_{h}\,\}$$

Similarly for features other than forwarders

Lemma $\nu h\,(h \triangleright k \mid A) \gtrsim A\{^{k}/_{h}\}$



– **Simple proof**
  * local property of the AM
  * up-to techniques applicable to expansion
    (ex: expansion up-to expansion)

Rigidity of expansion

- $P \lesssim Q$ says: P is **better at every step**
  - No, if P has some initial adminstrative work
- **Relations more flexible than $\lesssim$:** **[Pous 05,06]**
- **Example of application:** optimisations the AM

  [Hirschkoff, Pous, Sangiorgi, 05]
  - garbage collection of useless forwarders
  - remove chains of forwarders

Conclusions, part IV

- **$\lesssim$, or other relations:**
  - needed to control silent moves
  - allow us to reduce candidate relations only normal forms
- **$\lesssim$: nice mathematical properties, sometimes too rigid**

Problem 4: up-to in the weak case

a) Improvements of $\lesssim$

- better notion of “efficiency”
- more powerful up-to
  (goal: normal forms in candidate relations)

b) Composition of up-to techniques

- how can chaining be replaced?

Important! (practical relevance of **weak** bisimilarity)

Partial results: **[Pous 05,06]**

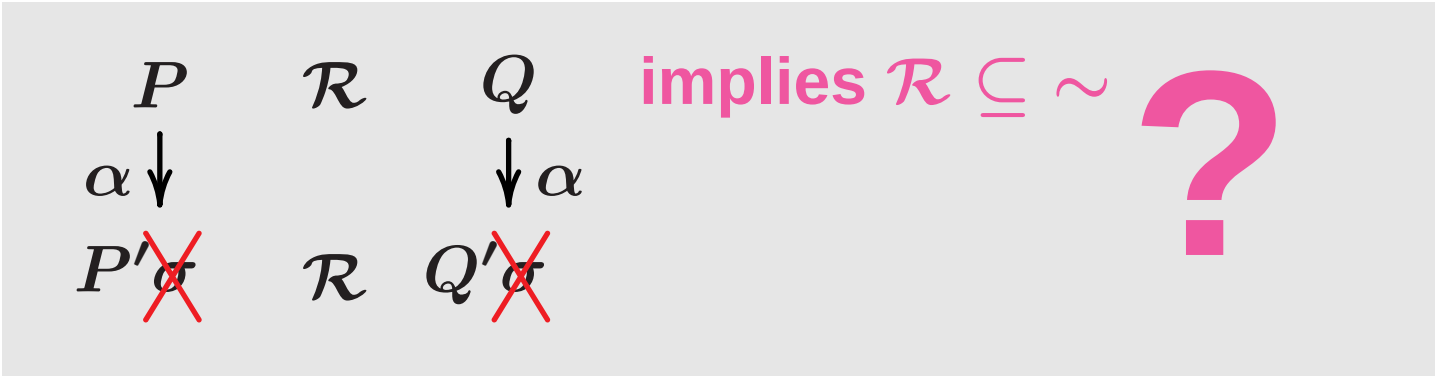
Problem 5: Mechanical verification

- **How can these enhancements be integrated in tools ?**

- **Partial results** [Hirschkoff]
  - theorem provers
  - automatic checking
  - Applied to **infinite-state** processes

Problem 6: Other primitive techniques

– Example: up-to substitutions sound in the π-calculus



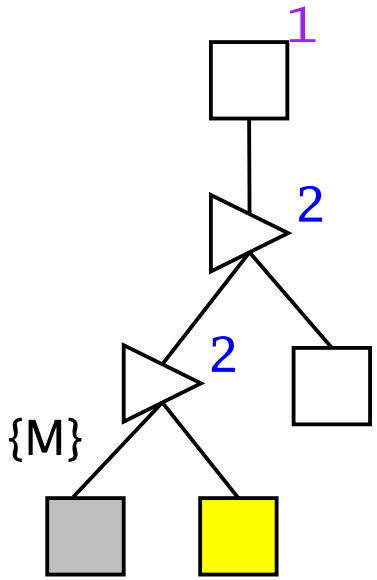
Rigidity of expansion

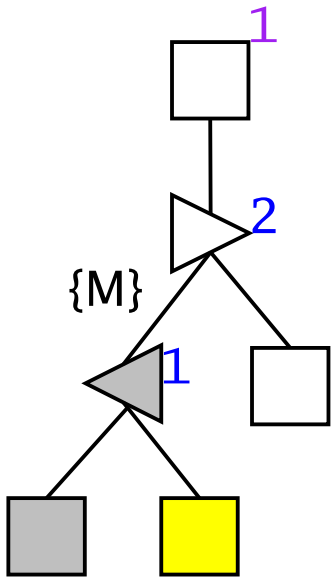
$P \lesssim Q$ says: P is better at **every** step

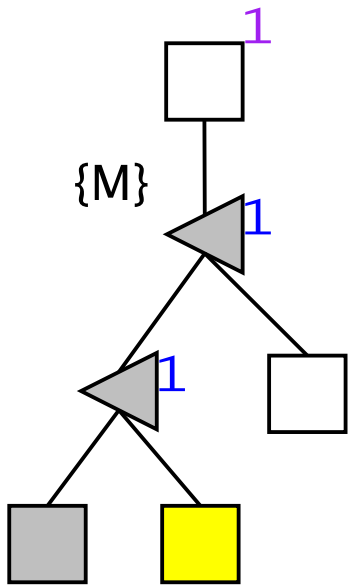
Example: optimise the AM [Hirschkoff, Pous, Sangiorgi, '05]

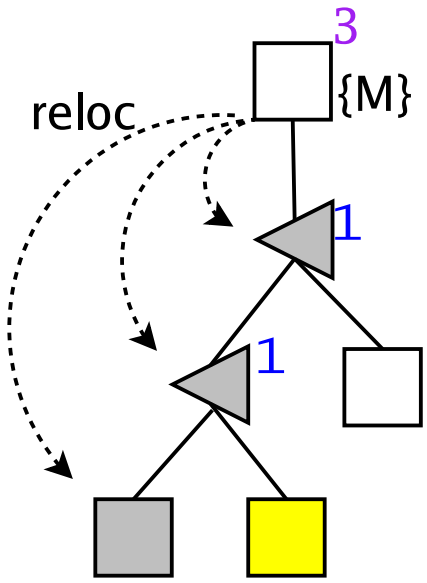
- **garbage collection of useless forwarders**
  - use counters in forwarders (= number of children)
- **remove chains of forwarders**
  - adapt Tarjan's union-find algorithm (**relocation**)

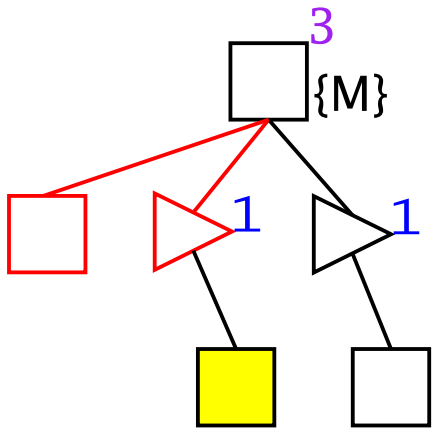
Relocation (cf: Tarjan sets)









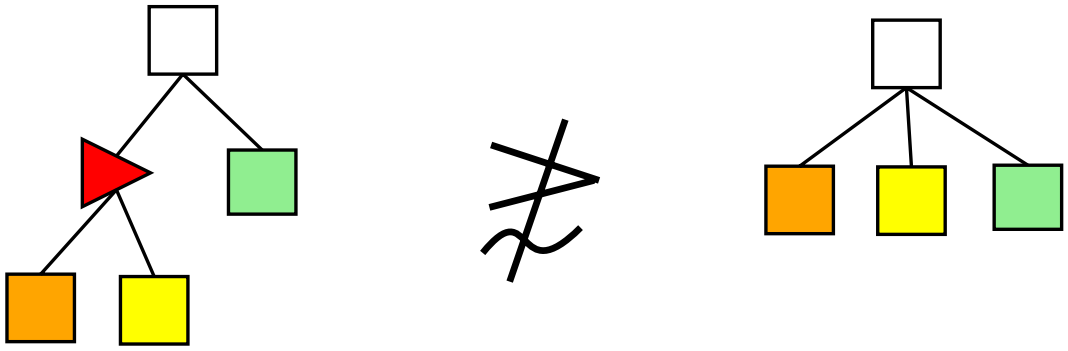


$[\![P]\!] \triangleq$ the original machine

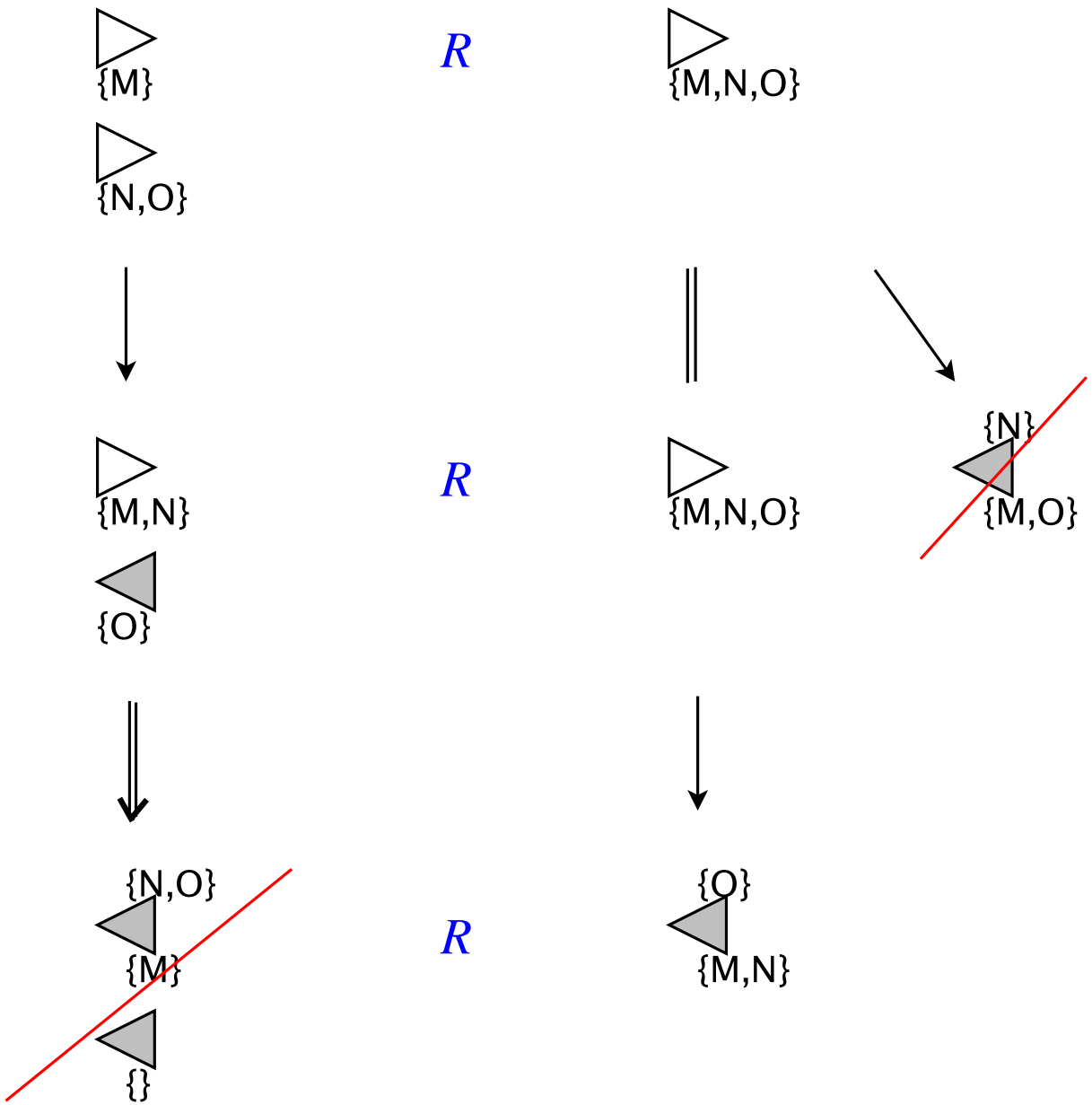
$[P] \triangleq$ the optimised machine

- $[P]$ **obviously better, but** $[P] \not\precsim [\![P]\!]$
  - initial adminstrative work, that only later pays off
- **Worst: in the optimised machine:**

$$\nu h\,(h \triangleright k \mid A) \not\succsim A\{^{k}/_{h}\}$$



$$\nu h\,(h \triangleright k \mid A) \;\not\gtrsim\; A\{^{k}/_{h}\}$$



Equivalence between the two machines

- Ideally, using $\mathcal{R} \triangleq \{([\![P]\!], [P])\}$ (normal forms)

  But $\mathcal{R}$ is not a bisimulation up-to expansion

- **Correctness proof in [Hirschkoff, Pous, Sangiorgi, '05]: a full bisimulation**

- **[Pous 05]: A proposal for relations more flexible than $\lesssim$**
  - Now $\mathcal{R}$ works
  - Define properties needed in a relation for the "up-to"

    Example: termination of the transitive closure
  - The relation need not be behaviourally interesting
  - Drawbacks: proving the conditions, compositionality